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# MATHCOUNTS®

## 2018
## ■ State Competition ■
## Sprint Round
## Problems 1−30

**HONOR PLEDGE**
I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature ______________________________ Date ____________

Printed Name ______________________________

School ______________________________

Chapter ______________________________

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

| Total Correct | Scorer's Initials |
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1. ______________ If $x \nabla y$ is defined as $(x + 1)(y - 1)$, what is $4 \nabla 3$?

2. __________ integers How many integers are equal to their own cube?

3. ______________ Four consecutive odd integers sum to 256. What is the least of the four integers?

4. __________ units The area of a square is, numerically, 5 times its perimeter. What is the greatest possible length of a side of the square?

5. ______________ If $f(x) = x + 2$, $g(x) = x^2 - 3$ and $h(x) = x - 5$, what is the value of $f(g(h(10)))$?

6. ______ baskets

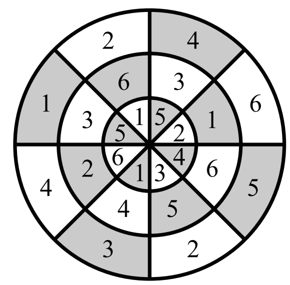
Steph is playing basketball with his brother. He scores 29 points on 12 successful baskets. Each basket was worth either 2 or 3 points. How many baskets worth 3 points did Steph make?

7. ______ chips

Alexandra plays poker with seven friends. Each of the eight players starts with 60 chips, but players take chips from one another as play progresses. The goal of the game is to win all of the chips. A player drops out of the game if she loses all her chips to other players in the game. After five players have dropped out of the game, what is the average number of chips each of the remaining players has?

8. ______

Setzer throws a dart that lands within one of the 24 numbered regions on the dartboard shown, and then rolls a standard six-sided die. What is the probability that the number of the region his dart hits is the same as the number he rolls on the die? Express your answer as a common fraction.




9. ______

The product 33 × 34 × 57 × 65 is divisible by the square of a prime number $p$. What is the value of $p$?

10. ______

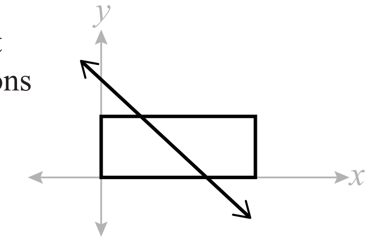
An ordinary 3-by-3 magic square contains every positive integer from 1 through 9, with one integer per cell, such that the sums of the numbers in each row, each column and each diagonal are the same. When the ordinary magic square shown is completed, what is the sum of all the possible values of $x$?

| | | |
|---|---|---|
| | $x$ | |
| | 5 | |
| | | $x+1$ |

11. ______________

The line $x + y = 5$ intersects a rectangle with vertices at (0, 0), (0, 3), (8, 0) and (8, 3), dividing it into two regions as shown. What is the ratio of the area of the smaller region to the area of the larger region? Express your answer as a common fraction.




12. ______________

If a runner who runs at a constant speed of $p$ miles per hour runs a mile in exactly $p$ minutes, what is the integer closest to the value of $p$?

13. ______________

The three-digit numbers C99, A6A, BC7 and B91 form an arithmetic sequence in this order. What is the value of $A^2 + B^2 + C^2$?

14. ______________

Let $x \star y = x^2 + xy$. If $a$ and $b$ are positive integers such that $a \star b = 9$ and $b \star a = 72$, what is the value of $\frac{a}{b}$? Express your answer as a common fraction.

15. ______________ mi/h

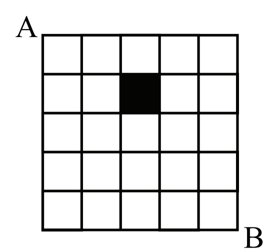
The team had an easy trip to the MATHCOUNTS contest, but a detour on the way home made the return trip take twice as long. If the new route home (including the detour) was 50% longer than the original trip, and the average speed returning was 10 mi/h slower, what was the average speed of the team going to the contest?

16. ______________ The orbital period of a planet is the time it takes to make one revolution around its sun. In a distant solar system, the giant planet Flion makes 5 orbits of its sun in the time it takes its planetary neighbor Reflurn to make 2 orbits. Flion makes 19 orbits of its sun in the time it takes the comet Hathov to make 3 orbits. What is the ratio of Hathov's orbital period to Reflurn's orbital period? Express your answer as a common fraction.

17. __________ points In eight games this season, Kelly's basketball team scored 22, 30, 33, 44, 50, 55, 61 and 66 points, respectively. They exactly tripled their opponent's score three times and exactly doubled their opponent's score three times. They lost two games by 4 points each. How many points did their opponents score altogether?

18. ______________ A 100-digit positive integer is divisible by 72. What is the greatest possible value of the sum of the number's digits?

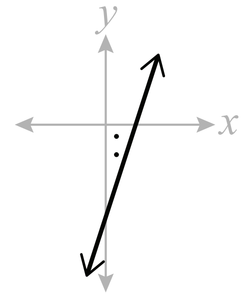
19. __________ ways In the figure, each segment between two adjacent vertices has length 1 unit. How many ways are there to go from A to B along a sequence of 10 segments without touching a side or vertex of the shaded square?




20. ______________ What is the sum of all the integer values of $x$ for which $|3x - 3| < 13$?

21. __________ points

The right triangle bounded by the $x$- and $y$-axes and the line $3x - y = 6$ contains 2 lattice points in its interior. How many lattice points will be contained in the interior of the triangle bounded by the $x$- and $y$-axes and the line $3x - y = 24$?




22. ______________

The base four representation of $p = 3 + \frac{0}{4} + \frac{2}{4^2} + \frac{1}{4^3}$ is $p = 3.021_4$. In base eight, $p = 3.AB_8$. What is the value of A + B?

23. ______________

Bryan visits a carnival booth where Carl shows him 10 boxes. Exactly one of the boxes contains a gold coin; the other boxes are empty. Bryan randomly takes one of the boxes, but he doesn't open it. Carl then opens five other boxes that he knows are empty and shows Bryan that they are empty. Carl then tells Bryan he can either keep his initially chosen box or return it and choose one of the remaining closed boxes instead. If Bryan chooses to return his box and choose another one instead, what is the probability Bryan will choose the box with the gold coin? Express your answer as a common fraction.

24. ______________

Ivory writes down three two-digit numbers, each with units digit 7. The tens digit of the product of these three numbers is 5. What is the tens digit of the sum of the three numbers?

25. ______________

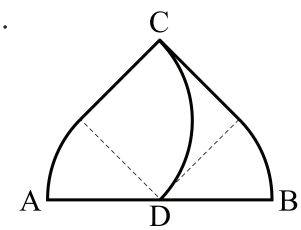
What is the probability that three randomly drawn real numbers between 0 and 1 have a sum less than 1? Express your answer as a common fraction.

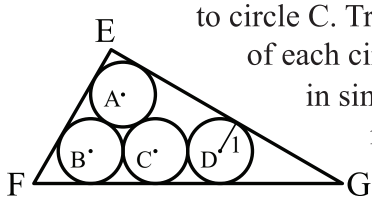
26. ______________ Kevin writes a sequence of numbers starting with 1, and repeatedly adding 1 until a multiple of 2 is reached. He then repeatedly adds 2 to this value until a multiple of 3 is reached, then adds 3 until he gets a multiple of 4, and so on. The first four terms are 1, 2, 4, 6. What will be the last term Kevin writes down before he adds 2000 for the first time?

27. ______________ Oliver rolls three fair standard six-sided dice. What is the probability that there is at least one pair of dice whose top faces sum to 6? Express your answer as a common fraction.

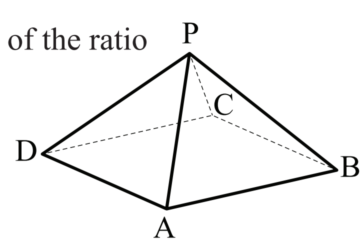
28. __________ percent A figure shown is bounded by segment AB and paths BC and CA, and is formed by seamlessly joining two 45-degree sectors of a circle of radius 1 unit along two sides of a unit square that has C and D as opposite vertices. A quarter-circular arc of radius 1 unit is drawn from C to D. What is the percent probability that a random point chosen on arc CD forms an acute triangle with points A and B? Express your answer to the nearest tenth.




29. ______________ Congruent, non-overlapping circles A, B, C and D are positioned in a plane, such that A, B and C are mutually tangent to each other, and circle D is tangent to circle C. Triangle EFG circumscribes the four circles as shown. If the radius of each circle is 1 meter, then the length of side FG can be expressed in simplest radical form as $a + b\sqrt{c}$ meters, where $a$, $b$ and $c$ are integers. What is the value of $a + b + c$?




30. ______________ In right pyramid PABCD, ABCD is a square with sides of length 2 and the distance from P to the center of square ABCD is 1. A plane intersects the pyramid at A and intersects edges PB, PC and PD at points B′, C′ and D′, respectively. If $\frac{B'B}{PB} = \frac{1}{4}$ and $\frac{D'D}{PD} = \frac{1}{5}$, what is the value of the ratio $\frac{C'C}{PC}$? Express your answer as a common fraction.



# Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

**Units of measurement are not required in answers, but they must be correct if given.** When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

**All answers must be expressed in simplest form.** A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where $a$ and $b$ are natural numbers and GCF($a$, $b$) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where $A$ and $B$ are algebraic expressions and $A$ and $B$ do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where $N$, $a$ and $b$ are natural numbers, $a < b$ and GCF($a$, $b$) = 1. Examples:

*Problem:* What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

*Problem:* What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

*Problem:* What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

*Problem:* What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

**Ratios should be expressed as simplified common fractions** unless otherwise specified. Examples:
*Simplified, Acceptable Forms:* $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

**Radicals must be simplified.** A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
*Problem:* What is the value of $\sqrt{15}\times\sqrt{5}$? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

**Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits.** The *only* exceptions to this rule are when $a$ is zero, in which case it may be omitted, or when $b$ and $c$ both are zero, in which case they both may be omitted. Answers in the form ($)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:
*Acceptable:* 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

**Do not make approximations for numbers** (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

**Do not perform any intermediate rounding** (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

**Scientific notation** should be expressed in the form $a \times 10^n$ where $a$ is a decimal, $1 \le |a| < 10$, and $n$ is an integer. Examples:
*Problem:* What is 6895 expressed in scientific notation? *Answer:* $6.895 \times 10^3$
*Problem:* What is 40,000 expressed in scientific notation? *Answer:* $4 \times 10^4$ *or* $4.0 \times 10^4$

**An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.**
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

**The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.**